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## High Performance Computing and Economic Scenario Generation: Integrating Expert Forecasts into Plane Price Modeling as an Example


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# High Performance Computing and Economic Scenario Generation: Integrating Expert Forecasts into Plane Price Modeling as an Example

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**Abstract.** The problem at hand is the integration of expert forecasts for plane prices into a fully calibrated basic economy. The economy is simulated through an Economic Scenario Generator (ESG), which includes macroeconomic processes, interest rate term structures, etc.. By defining the available best-case, worst-case, and mid-case forecasts to correspond to the 95%, the 50% and the 5% quantiles of the plane price distribution, one could describe the problem with the following optimization setting:

$$\min_{(\beta_c,\alpha_{c,i},\delta_{\mathbf{c}}{}',\sigma_{c,i})}\{\| \left(Q\left((\widetilde{I}_{T,S})\odot\widetilde{P}^R_{c,i,T,S}(\beta_c,\alpha_{c,i},\delta'_{\mathbf{c}},\gamma'_{\mathbf{c}},\sigma_{c,i}),\mathbf{q}\right)-\widetilde{FM}\right)\odot W\|^2_F\}$$

The tilded matrices represent simulation results, i.e. they have the dimension timesteps $T$ and scenarios $S$. The function $Q(\widetilde{M}_{T,S},\mathbf{q}):\mathbb{R}^{T\cdot S}\times[0,1]^q\rightarrow\mathbb{R}^{T\cdot q}$ is mapping a matrix $\widetilde{M}_{T,S}$ of simulated scenarios with dimension $(T\times S)$ onto each timestep's quantiles $\mathbf{q}$ , resulting in a matrix of dimension $T\times q$. FM is the $(T\times q)$ matrix of expert forecasts, and W is a $(T\times q)$ weighting matrix. $\|\|_F$ denotes the Frobenius norm,[1] and $\odot$ is the element-wise multiplication. The economy simulation is computation-intensive, for which we take benefit of using GPGPU techniques. The optimisation part is also a high-dimensional computation-intensive problem, for which we use a natural computing approach using Differential Evolution.

**Keywords:** Economic Scenario Generation, Plane Price Modelling, Expert Forecasts, GPU Computation, Differential Evolution

## 1 Basis Economy

For the comprehensive modeling of plane prices we use the risklab Economic Scenario Generator (ESG)[2] as the basic theoretical framework. This model builds

[1] Compare e.g. [3].
[2] Compare [11].

on fundamental macroeconomic factors to describe the evolution of interest rates and equities. Using a cascade structure, the model captures the long-term economic relationships while allowing for short-term deviations. This structural approach allows for an integrated modeling of financial markets to obtain economically meaningful and consistent scenarios. We assume an arbitrage-free, frictionless financial market in continuous time $t \in [0, T^*]$, where $T^*$ is the end of the given time horizon and the uncertainty in the market is described by the complete filtered probability space $(\Omega, \mathcal{F}, F, \mathbb{P})$. Details on this framework can be found in [10]. The numeraire is represented by a non-defaultable money market account defined by $P(0,t) = \int_0^t e^{r^N(s)} ds$, the process $\{r^N(t)\}_{t\in[0,T^*]}$ represents the nominal short rate. We assume the existence of a probability measure $Q$ equivalent to $\mathbb{P}$, under which all discounted price processes of the financial market under consideration are martingales. Onto this foundation, we impose the cascade structure mentioned before and explained in the following, to incorporate long-term economic dependencies.

The first cascade of the model comprises an inflation process $\{i(t)\}_{t\in[0,T^*]}$ and an economic growth process $\{w(t)\}_{t\in[0,T^*]}$, which are both modeled with Vasicek processes, introduced in [9]. Under the equivalent martingale measure $Q$ they are specified as follows:

$$di(t) = [\theta_i - \hat{a}_i \cdot i(t)]dt + \sigma_i dW_i^Q(t), \tag{1}$$

$$dw(t) = [\theta_w - \hat{a}_w \cdot w(t)]dt + \sigma_w dW_w^Q(t), \tag{2}$$

with the positive real numbers $\theta_i, \theta_w, \hat{a}_i, \hat{a}_w, \sigma_i, \sigma_w$ and the independent standard Brownian motions $W_i^Q$ and $W_w^Q$. The third risk factor from the first cascade is the real oil price, which is directly relevant for plane price modeling. We assume the real oil price $P_o^R$ to follow a geometric Ornstein-Uhlenbeck process under the real measure $\mathbb{P}$, which is specified as follows:[3]

$$dP_o^R(t) = [\theta_o - a_o \cdot \log P_o^R(t)]P_o^R(t)dt + \sigma_o dW_o(t), \tag{3}$$

with $\theta_o$,$a_o$ and $\sigma_o$ as positive real numbers.

The following second cascade contains the real and nominal interest rate processes. The real short rate process $\{r(t)\}_{t\in[0,T^*]}$ is modeled with a two factor Hull White model. Its dynamics are specified as:

$$dr(t) = [\theta_r(t) + b_{rw} \cdot w(t) - \hat{a}_r \cdot r(t)]dt + \sigma_r dW_r^Q(t), \tag{4}$$

with $b_{rw}$, $\hat{a}_r$, and $\sigma_r$ being positive real numbers, $\theta_r$ being a time dependent deterministic function and the standard Brownian motion $W_r^Q$ being independent of the Brownian motions mentioned before. The nominal short rate $\{r^N(t)\}_{t\in[0,T^*]}$ comprises real short rate and inflation short rate, and is obtained as the sum of real short rate and inflation, i.e.:

$$r^N(t) = r(t) + i(t). \tag{5}$$

[3] See[2]for more information on the geometric Ornstein-Uhlenbeck process.

The term structure of nominal interest rates can then be derived by the zero-coupon bond prices obtained from this setting. [11] show, that under the presented set of assumptions the price of a zero-coupon bond with maturity $t < T \leq T^*$ is given by:

$$P(t,T) = e^{(A(t,T)-B(t,T)r(t)-C(t,T)i(t)-D(t,T)w(t))}, \tag{6}$$

where

$$\begin{aligned}
B(t,T) &= \tfrac{1}{\hat{a}_r}(1-e^{(-\hat{a}_r(T-t))}),\\
C(t,T) &= \tfrac{1}{\hat{a}_i}(1-e^{(-\hat{a}_r(T-t))}),\\
D(t,T) &= \tfrac{b_{rw}}{\hat{a}_r}\cdot\left(\tfrac{1-e^{(-\hat{a}_r(T-t))}}{\hat{a}_w}+\tfrac{e^{(-\hat{a}_w(T-t))}-e^{(-\hat{a}_r(T-t))}}{\hat{a}_w-\hat{a}_r}\right),\\
A(t,T) &= \int_t^T(\tfrac{1}{2}(\sigma_r^2 B(l,T)^2+\sigma_i^2 C(l,T)^2+\sigma_w^2 D(l,T)^2)-\theta_r(l)B(l,T)\\
&\quad -\theta_i C(l,T)-\theta_w D(l,T))dl.
\end{aligned}$$

The model equations can be derived under the real measure $\mathbb{P}$ instead of the equivalent martingale measure $Q$ using Girsanov's Theorem, by replacing $W_i^Q$, $W_w^Q$, $W_r^Q$ with the independent standard Brownian motions $W_i$, $W_w$, $W_r$ and using the parameters $a_i = \hat{a}_i - \lambda_i\sigma_i^2$, $a_w = \hat{a}_w - \lambda_w\sigma_w^2$, and $a_r = \hat{a}_r - \lambda_r\sigma_r^2$. The parameters $\lambda_i$, $\lambda_w$, and $\lambda_r$ are obtained by the change of measure, as shown e.g in [7]. In addition, we assume the Brownian motion $W_o(t)$ to be correlated with $W_w(t)$ with a constant correlation coefficient $\rho > 0$. As can be seen, the term structure of interest rates is driven by the real short rate process, as well as the underlying macroeconomic factors of economic growth and inflation rates. The third cascade contains equity assets. The equity prices $\{S_t^E\}_{t\in[0,T^*]}$ are driven by the following dynamics:

$$dS^E(t) = \left[\alpha_E + b_{Er}r(t) - b_{Ei}i(t) + b_{Ew}w(t)\right]S^E(t)dt + \sigma_E S^E(t)dW_E(t), \tag{7}$$

where $b_{Er}$, $b_{Ei}$, $b_{Ew}$, and $\sigma_E$ are positive real numbers, $\alpha_E \in R$ and $W_E(t)$ is a standard Brownian motion, independent of those mentioned above.

## 2 Plane Price Model

On the basic economy just described, we will impose our model of the plane price process. As plane prices are dependent on macroeconomic factors, whereas macroeconomic factors do not depend on plane prices, we follow a two-step procedure to simulate plane prices. In a first step, we simulate the basic economy using the risklab ESG, and then impose the plane price process onto the calibrated basic economy. Using the methods of dynamic panel data analysis,[4] one can identify the following model describing the real prices in year $t$ of a plane

[4] For a detailed exposition of panel data analysis methods, compare [4]. For log-linear plane price modeling and the inclusion of age dependency compare e.g. [6].

belonging to class $c$[5] and built in year $i$ under $\mathbb{P}$:[6]

$$\begin{aligned} P^R_{c,i}(t) = \exp(&\beta_c + \alpha_{c,i} + \rho_c \cdot \log(1 + P^R_{c,i}(t-1)) \\ &+ \delta_{\mathbf{c}}{}' \cdot \mathbf{A_{c,i}(t)} + \gamma_{\mathbf{c}}{}' \cdot \mathbf{M(t)} + \sigma_{c,i}\epsilon_{c,i}(t)) - 1, \end{aligned}$$

with the vectors $\mathbf{A_{c,i}(t)}$ and $\mathbf{M(t)}$ defined as:

$$\mathbf{A_{c,i}(t)} := \left(\log(1 + \mathrm{age}_{c,i}(t))),\ (\log(1 + \mathrm{age}_{c,i}(t)))^2,\ (\log(1 + \mathrm{age}_{c,i}(t)))^3\right)',$$

$$\mathbf{M(t)} := \left(w(t),\ \log(1 + P^R_o(t)),\ \frac{1}{P(t, T_M)},\ \log \frac{S^E(t)}{S^E(t-1)}\right)',$$

and the parameter vectors $(\beta_c,\ \alpha_{c,i}) \in \mathbb{R}^2$, $\delta_{\mathbf{c}} \in \mathbb{R}^{\mathbf{3}}$and $\gamma_{\mathbf{c}} \in \mathbb{R}^4$. As introduced before, $w_t$ is GDP-growth, $\mathrm{age}_{c,i}(t)$ is the age of the plane, $P^R_o(t)$ is the real oil price and $P(t, T_M)$ is the price of a zerobond with a notional of 1 currency unit and one year to maturity, and the residual noise $\epsilon_{c,i}(t) \sim N(0, 1)$.

The nominal plane price $P^N_{c,i}(t)$ can then be obtained by multiplying the real price $P^R_{c,i}(t)$ with the inflation index $I(t)$:

$$P^N_{c,i}(t) = P^R_{c,i}(t) \cdot I(t), \tag{8}$$

where $I(t)$ is the value of the inflation index[7] generated by $\{i(t^*)\}_{t^* \in [0,t]}$ at time $t$. With this model, macroeconomic variables obviously have an impact on plane prices, but not vice versa.

## 3 Simulation and calibration

The problem we are facing now is the integration of the plane price model and expert forecasts for plane prices into the calibrated basic economy. As there is only a one-way dependence, i.e. plane prices depend on macroeconomic factors, but not vice versa, the first step of integration is done simply by calculating plane prices given the exogenous factor realizations from the basic economy. However, also the given expert forecasts on plane prices need to be considered. Usually, one is given best-case, worst-case, and mid-case forecasts for plane prices. One approach is then to define these three cases to correspond to specific quantiles of the plane price distribution, e.g. 5% for the worst case, 50% for the mid case and 95% for the worst case. Following this approach, one could describe the problem of integrating the expert forecasts generally by the following optimization setting:[8]

$$\min_{(\beta_c, \alpha_{c,i}, \delta_{\mathbf{c}}{}', \sigma_{c,i})} \{\| \left(Q\left((\widetilde{I}_{T,S}) \odot \widetilde{P}^R_{c,i,T,S}(\beta_c, \alpha_{c,i}, \delta_{\mathbf{c}}{}', \gamma'_{\mathbf{c}}, \sigma_{c,i}), \mathbf{q}\right) - FM\right) \odot W\|^2_F\}. \tag{9}$$

[5] E.g. widebody, narrowbody, etc.

[6] For the ease of exposition, we present a reduced model. The generalized model considers several economies with their proper economic processes and linking exchange rates.

[7] Starting with $I(0) = 1$.

[8] In this optimisation, we consider only one build-year. Thus, we set $\alpha_{c,i} = 0$.

The tilded matrices represent simulation results, i.e. they have the dimension timesteps $T$ and scenarios $S$. The matrix $\widetilde{P}^R_{c,i,T,S}$ is for instance composed of the components:

$$P^R_{c,i}(t,s) = \exp(\beta_c + \alpha_{c,i} + \rho_c \cdot \log(1 + P^R_{c,i}(t-1,s)) \\ + \delta_{\mathbf{c}}{}' \cdot \mathbf{A_{c,i}(t)} + \gamma_{\mathbf{c}}{}' \cdot \mathbf{M(t,s)} + \epsilon_{c,i}(t,s)) - 1.$$

The function $Q(\widetilde{M}_{T,S}, \mathbf{q}) : \mathbb{R}^{T \cdot S} \times [0,1]^q \to \mathbb{R}^{T \cdot q}$ is mapping a matrix $\widetilde{M}_{T,S}$ of simulated scenarios with dimension $(T \times S)$ onto each timesteps' quantiles $\mathbf{q}$ , resulting in a matrix of dimension $T \times q$. FM is the $(T \times q)$ matrix of expert forecasts, and W is a $(T \times q)$ weighting matrix[9]. $\|\|_F$ denotes the Frobenius norm,[10] and $\odot$ is the element-wise multiplication.

In our case, best, worst and mid case forecasts are given. So we set $q = 3$ and $\mathbf{q} = (0.05, 0.5, 0.95)'$. Note that the optimization is only over the parameter vector $(\beta_c, \alpha_{c,i}, \delta_{\mathbf{c}}{}', \sigma_{c,i})$. This is due to the fact that the parameter vector $\gamma_{\mathbf{c}}$ represents economic relationships, i.e. sensitivities of real plane prices to macroeconomic variables and is estimated on empirical data.[11] Thus, this vector should not be altered in the course of the optimization. However, the parameter vector $(\beta_c, \alpha_{c,i}, \delta_{\mathbf{c}}{}')$ controls a deterministic evolution of plane prices over time, and is therefore predestined to incorporate the expert forecasts. The parameter $\sigma_{c,i}$ incorporates the experts opinion on quantile spread. Therefore, we face the large-scale unrestricted optimization problem described above, which we solve using differential evolution.

The optimization result can be seen in Figure 1 exemplarily for a one plane-class. The bars show each year's nominal plane price distribution quantiles, the dots show the median. The red lines indicate the given expert forecasts considered in the optimization. The image shows that the plane price simulation using the optimized parameters matches well the given forecasts.

## 4 GPGPU Computation

As simulating a large number of scenarios for the Economic Scenario Generator could be quite computation-intensive, we are making use of the GPGPU techniques to harness the computation intensivity of the basic economy simulation. The corresponding stochastic processes simulated using GPGPUs are illustrated in Figure 2, ordered consistently with the cascading structure of the risklab ESG.

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[9] One has to take into consideration that the components of the weighting matrix will be squared when applying the Frobenius norm. As a result, corresponding transformations should be applied to the weighting matrix when needed.

[10] Compare e.g. [3].

[11] The parameters were in fact estimated on historical data using a generalized method of moments following Arrelano and Bover, which can be found e.g. in [4], p. 53f.

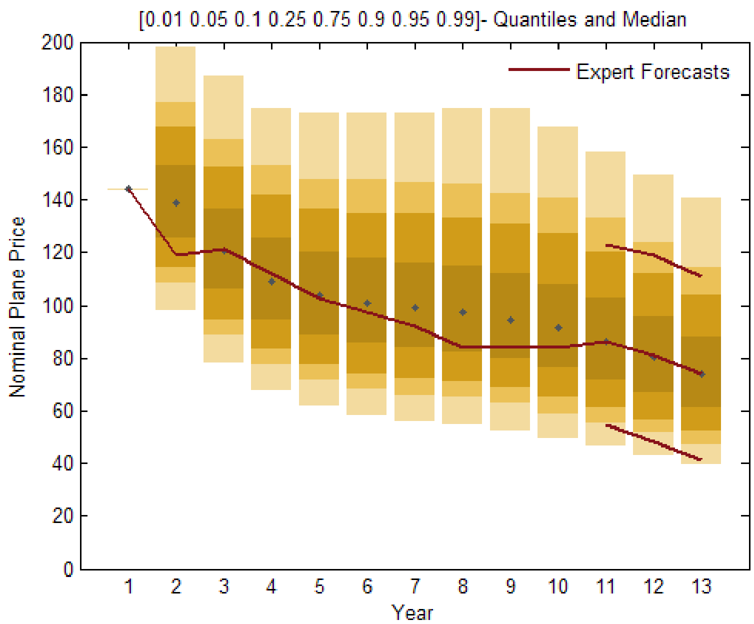


**Fig. 1.** Simulated Quantiles and Expert Forecats

For our purposes, we are using the NVIDIA Quadro FX 580 GPU, which comes equiped with 32 processing cores.

GPUs are computation devices that are capable of running a very large number of threads simultaneously. A key requirement to the efficient use of GPUs for computation is independence of the running threads. This means the running threads are not allowed to interact with each others and will be operating on different data. As a result, data independence of the executing threads is also a requirement. However, threads are allowed to share data but without I/O flow dependencies. GPUs also have the advantage of leviating the use of the main CPU since GPUs operate as coprocessors for the latter. In the context of GPGPU, the main CPU is also known as the **host**, and the GPU devices are known as the **device**. For these two computation devices to coexist without intevening with each other's work, they would need to maintain separate memory spaces, entitled the **host memory** and the **device memory** respectively. In addition, to best utilize the computation resources of the GPUs, a programming interface is provided from NVIDIA entitled CUDA (Compute Unified Device Architecture).

So, for our application, the simulation of the underlying risk factors representing the basic economy as well as the plane price process fit nicely to the target applications of GPGPU techniques. The different processes are data parallel and computation-intensive. For this purpose, we had to separate the different process Stochastic Differential Equations (SDEs) into CUDA functions that are followingly compiled into the instruction set of the GPU device. What we get after compiling these CUDA functions against the device instruction set are also called in CUDA parlance **kernels** and are then downloaded to the GPU device for execution.

The simulated scenarios run independently of each other and have no data dependencies. Since what differentiates the scenarios is the diffusion part of their stochastic differential equation, which stems from the dynamics of the corresponding Brownian motion, generating random numbers is a critical part in our simulation. In addition, the dynamics driving the plane prices and those driving the growth of the economy are positively correlated ($\rho > 0$).

As as result, we choose to generate the needed random numbers in parallel making use of a parallel implementation of the **Mersenne Twister** (MT) Pseudo-Random Number Generator (PRNG) that leviates the inherent serial behavior of the Mersenne Twister and allows threads to generate random numbers in parallel, with good statistical properties. This implementation of the Mersenne Twister comes from the original founders of the MT and is called DCMT (Dynamic Creation of Mersenne Twisters). The generated random numbers are then adapted to relevant correlations and made available to the different kernels to run on on a full-storage-method approach.

## 5 Differential Evolution

Differential Evolution (DE) is a vector-population-based stochastic optimization method that was introduced to the public in 1995. It has been widely used since then and in various domains of applications. Pointers for further reference in this respect can be found in [5], [8], or [1], among others. Figure 3 shows through the help of a flow chart the underlying idea and steps behind the Differential Evolution (DE) algorithm. This chart is adapted to our problem at hand from [5]. To simplistically convey the general idea of this algorithm, we are limiting the size of the population of our samples to 6. Our parameter vector for the optimisation problem in Setting 9 is composed of 5 parameters[12], namely $(\beta_c, \sigma_{c,i}, \delta_{c,1}, \delta_{c,2}, \delta_{c,3})$ with $\delta'_c = (\delta_{c,1}, \delta_{c,2}, \delta_{c,3})$. The values for the different parameters are random for the sake of clarification and do not represent the final parameters.

At each iteration, DE compares the current population to a competent population. A population is the number of parameter vectors assessed at the given iteration. Hence, at a given iteration $k$, DE takes the best population from the last iteration and for each individual in the population (target individual), it constructs a competent individual by taking the difference of two randomly chosen individuals and adds it to either the target individual or another randomly selected individual from the population. Then, a single candidate vector is chosen from the two individuals uniformally based on a crossover bound (CR) probabililty. The probabilistic approach for selecting the final vector based on the CR

[12] Remember that $\alpha_{ci} = 0$.

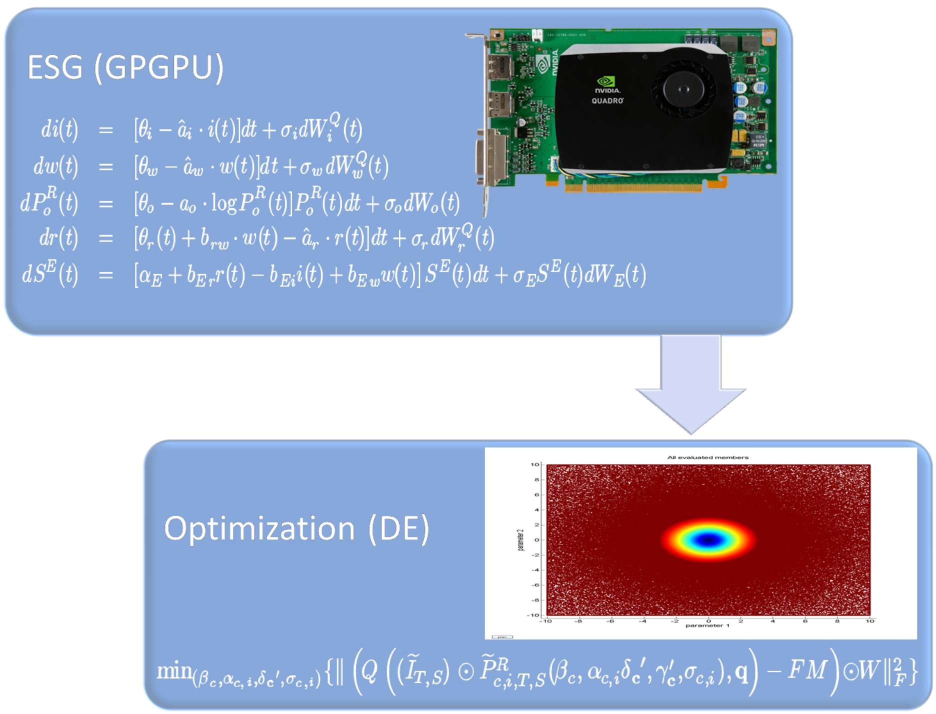


**Fig. 2.** GPGPU Computation and Differential Evolution Applied to Our Process.

probability is applied per parameter value individually (see Figure 3). The selected candidate is then evaluated and used for the next iteration. The algorithm starts by randomly selecting a starting vector from the defined boundaries of the optimization problem.

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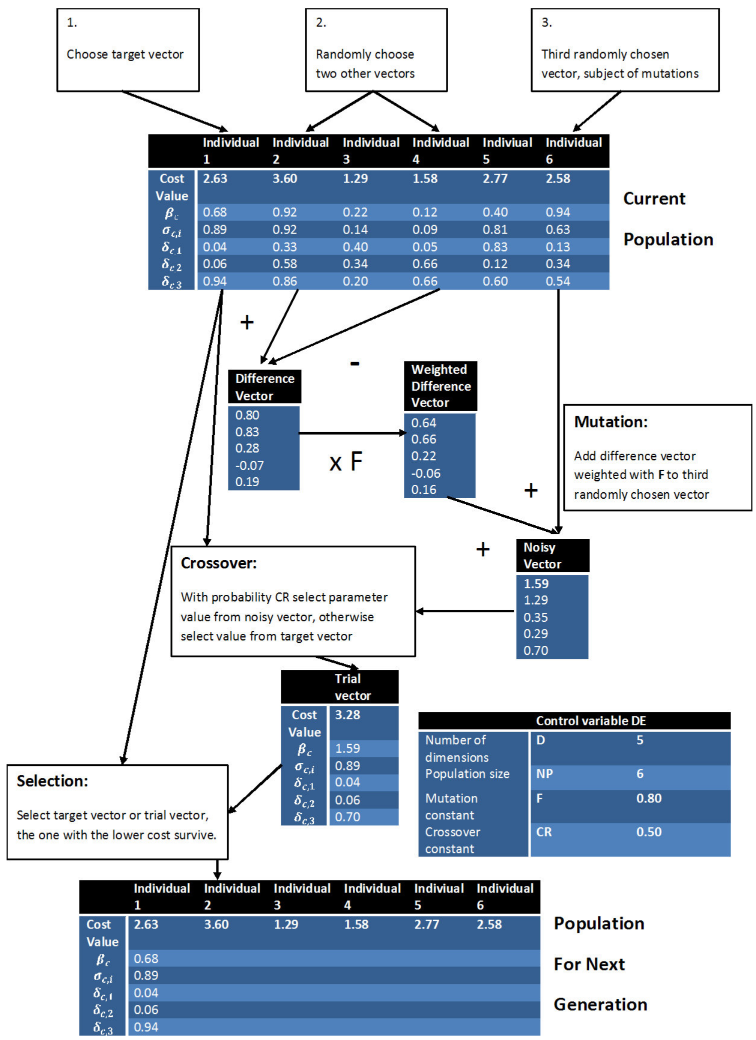


**Fig. 3.** Pictorial Workings of DE (from [5])